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Convergence of the Subsonic Doublet Point Method

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Nomenclature

C_l	=	lift coefficient
C_m	=	pitching moment coefficient
h	=	plunge amplitude
k	=	reduced frequency based on wing semispan
k_r	=	reduced frequency based on mean wing semichord
M	=	Mach number
n	=	number of boxes in aerodynamic model
n_y	=	number of spanwise strips on wing
s	=	wing semispan

Introduction

THE subsonic doublet point method[1] (DPM) is a simpler method compared to the subsonic doublet lattice method[2–4] (DLM) and offers computational efficiency at the cost of physical detail. In particular, it avoids the necessity of spanwise integration of the kernel functions but loses the representation of sweep by individual aerodynamic boxes. However, both methods should converge to the same result in the limit of infinitely small aerodynamic boxes. It was stated in Ref. 5 that the test cases used in Ref. 1 did not address the differences between the DPM and the DLM adequately, particularly with respect to sweep. The present work aims to give an indication of the effect of how this property is represented in the DPM compared to the DLM using some of the test cases suggested in Ref. 5. Note that the convergence of the subsonic DLM is almost linear with box size when the chordwise and spanwise paneling is refined simultaneously.[6] The same property is shown to exist for the doublet point method. Three test cases are considered: the AGARD wing and tail, a 70-deg delta wing, and a circular wing.

AGARD Wing and Tail

The unsteady lift coefficient of the AGARD wing and tail in plunge at $M = 0.8$ and $k_r = 1.2$ was calculated using the DLM and DPM and the same series of successively finer grids as was used in Ref. 6. The aerodynamic grids were generated using the minimum number of boxes while satisfying restraints on maximum box aspect ratio and maximum box chord. Box aspect ratios of 8, 2, and 0.5 were specified. A series of successively finer grids were generated for each aspect ratio by specifying a range of decreasing box chords. Figure 1 shows the convergence histories of the DLM and DPM results. The results are plotted against $1/\sqrt{n}$, which is proportional to linear box size. Both the DPM and DLM results converge along approximately straight lines with different slopes to practically the same value.

The variation between the results for the different box aspect ratios is greater for the DPM than for the DLM. However, the DLM results for a particular grid is not always closer to the converged value than the DPM result, in particular for the Æ 0.5 series of grids. In the case of the Æ 8 series of grids, which may be inappropriate for the DPM considering its single point representation, the convergence behavior of the DPM is not as linear as either the corresponding DLM results or the DPM results for the other series.

70 Degree Delta Wing

Figure 2 shows the convergence history of the pitching moment coefficient of a 70-deg delta wing pitching about the root midchord at $k_r = 2$ and $M = 0.8$. The same paneling schemes that were used in Ref. 7 are used here. The first series of paneling schemes is (span × chord) 10×10, 20×20, 30×30, 40×40, 50×50, 60×60, 70×70, and 80×80. The second series is 5×20, 10×40, 15×60, 20×80, 25×100, 30×120, 35×140, and 40×160, and the third series is 5×50, 10×100, 15×150, 20×200, and 25×250. Equally spaced divisions were used for both the spanwise and the chordwise paneling. The same observations as for the AGARD wing and tail apply.

Circular Wing

Figures 3 and 4 show the unsteady lift and pitching moment coefficients, respectively, for a circular wing pitching about its midchord

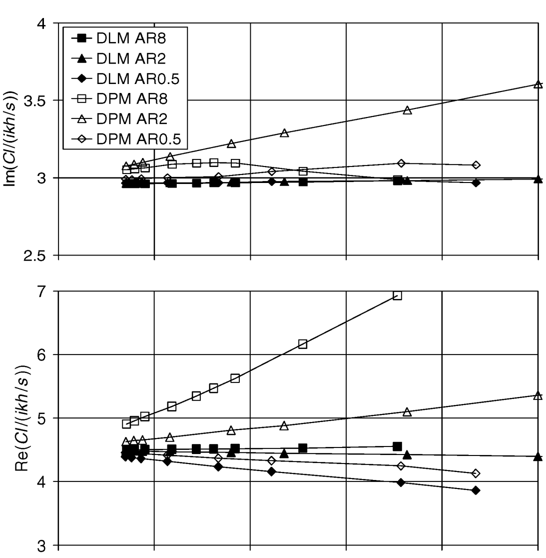

Fig. 1 AGARD wing and tail unsteady lift coefficient at $M = 0.8$ and $k_r = 1.2$.

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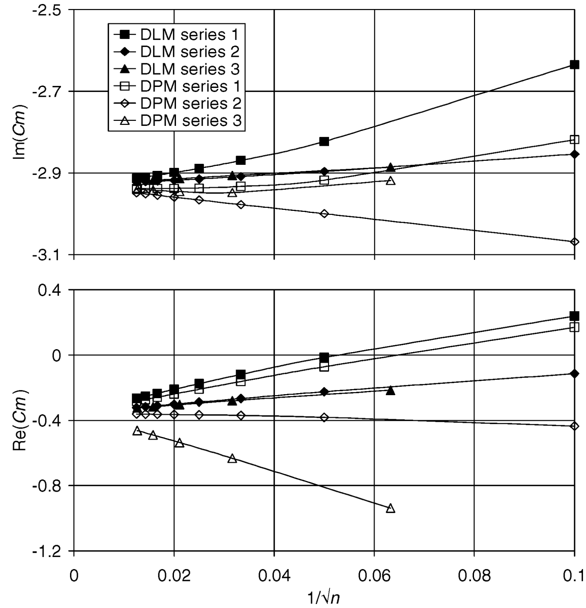

Fig. 2 Unsteady pitching moment coefficient for the 70-deg delta wing.

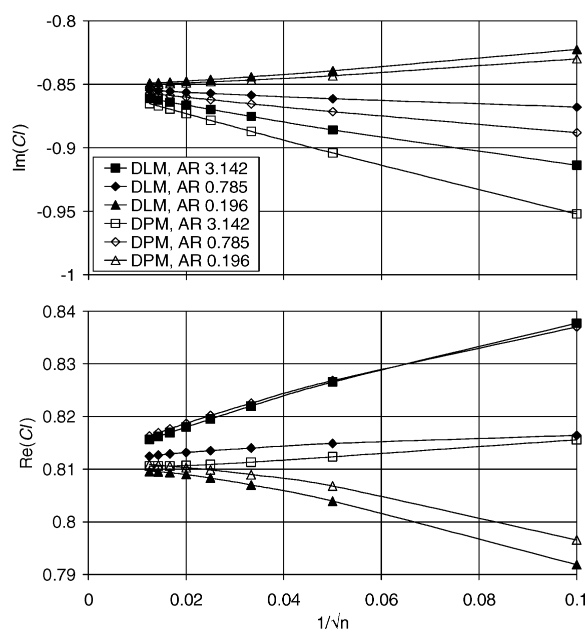

Fig. 3 Unsteady lift coefficient for the circular wing.

at Mach 0.0 and $k = 1.0$. The wing was divided into spanwise strips according to

$$y_i = r \sin[\pi i/(2n_y + 1)]$$

for $i = 0, \ldots, n_y$ and where r is a radius chosen so that the area of the grid is equal to the area of a unit circle. Each spanwise strip was divided into equally spaced chordwise boxes. Three series of grids were used, namely, (span × chord) 20 × 5, 40 × 10, 60 × 15, 80 × 20, 100 × 25, 120 × 30, 140 × 35, and 160 × 40 (AR 0.196); 10 × 10, 20 × 20, 30 × 30, 40 × 40, 50 × 50, 60 × 60, 70 × 70, and 80 × 80 (AR 0.785); and 5 × 20, 10 × 40, 15 × 60, 20 × 80, 25 × 100, 30 × 120, 35 × 140, and 40 × 160 (AR 3.142). The series are denoted by the limiting value of the box aspect ratio as the grid is refined.

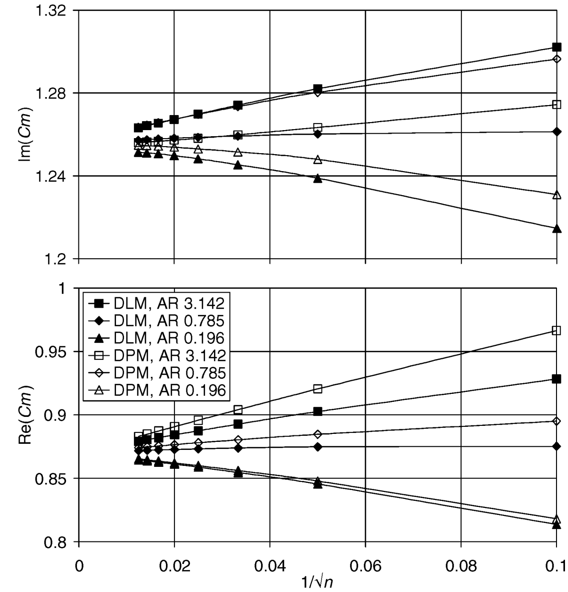

Fig. 4 Unsteady pitching moment coefficient for the circular wing.

The convergence of both the DLM and DPM results is regular, with a similar spread of values for the two methods.

Conclusions

The DPM and DLM converge to the same values for the overall coefficients in the examples considered here. As expected the spread of values over the range of grids considered is generally smaller for the DLM than for the DPM, but by a much smaller margin than would be expected considering the simplicity of the methods.

The regular convergence behavior of both the DLM and DPM when the spanwise and chordwise paneling is refined simultaneously suggests that the results can be extrapolated to $1/\sqrt{n} = 0$ to obtain an estimate of the fully converged result. However, note that the nonconverged results for cases involving control surface deflections are sometimes closer to reality than fully converged results. Another application of this property is that it provides an estimate of the nonconvergence error: The nonconvergence error for a particular paneling scheme is approximately equal to the difference between the results for that paneling scheme and the results for one with half as many chordwise boxes and half as many spanwise strips.

The present study only considered the convergence behaviour of the DLM and DPM for regular grids, that is, without abrupt changes in box size. Although similar trends are expected for grid refinement, the tolerance of the two methods to abrupt changes in aerodynamic box size as may be required for control surfaces is a matter for further investigation.

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